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COMMON FACTORS OF K-FIBONACCI-LIKE, K-FIBONACCI AND K-LUCAS NUMBERS

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ABSTRACT

The Fibonacci sequence is famous for possessing wonderful and amazing properties. In this paper, we present generalized identities involving common factors of k-Fibonacci-Like, k- Fibonacci and k-Lucas numbers and related identities. Binet's formula will employ to obtain the identities.

KEYWORDS: K-Fibonacci-Like Numbers, K-Fibonacci Numbers And K-Lucas Numbers, Binet's Formula

INTRODUCTION

It is well-known that the Fibonacci sequence is most prominent examples of recursive sequence. The Fibonacci sequence is famous for possessing wonderful and amazing properties. Fibonacci numbers are a popular topic for mathematical enrichment and popularization. The Fibonacci appear in numerous mathematical problems. Fibonacci composed a number text in which he did important work in number theory and the solution of algebraic equations. The book for which he is most famous in the "Liber abaci" published in 1202. In the third section of the book, he posed the equation of rabbit problem which is known as the first mathematical model for population growth. From the statement of rabbit problem, the famous Fibonacci numbers can be derived,

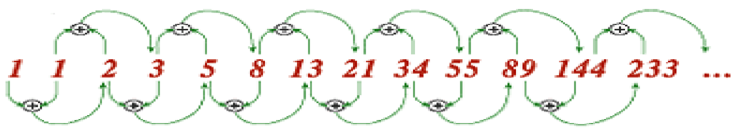

Figure 1

This sequence in which each number is the sum of the two preceding numbers has proved extremely fruitful and appears in different areas in Mathematics and Science.

The Fibonacci sequence, Lucas sequence, Pell sequence, Pell-Lucas sequence, Jacobsthal sequence and Jacobsthal-Lucas sequence are most prominent examples of recursive sequences.

The Fibonacci sequence [8] is defined by the recurrence relation

$$F_k = F_{k-1} + F_{k-2}\ ,\ k \geq 2\ \ with\ F_0 = 0,\ F_1 = 1 \tag{1.1}$$

The Lucas sequence [8] is defined by the recurrence relation

$$L_k = L_{k-1} + L_{k-2}\ ,\ k \geq 2\ \ with\ L_0 = 2,\ L_1 = 1 \tag{1.2}$$

The second order recurrence sequence has been generalized in two ways mainly, first by preserving the initial conditions and second by preserving the recurrence relation.

Kalman and Mena [5] generalize the Fibonacci sequence by

$$F_n = aF_{n-1} + bF_{n-2}\ ,\ n \geq 2 \quad with \quad F_0 = 0,\ F_1 = 1 \tag{1.3}$$

Horadam [1] defined generalized Fibonacci sequence by

$$H_n = H_{n-1} + H_{n-2},\ n \geq 3\ with\ H_1 = p,\ H_2 = p + q \tag{1.4}$$

where p and q are arbitrary integers.

Panwar, Rathore and Chawla [18] introduced the k-Fibonacci-Like sequence and proved some related identities. For any positive real number k, the k-Fibonacci-Like sequence $\{S_{k,n}\}$ is defined by the recurrence relation

$$S_{k,n+2} = kS_{k,n+1} + S_{k,n}\ ,\ n \geq 0\ \ with\ S_{k,0} = 2,\ S_{k,1} = 2k \tag{1.5}$$

This sequence contains features both of the k-Fibonacci sequence and the Fibonacci-Like sequence.

PRELIMINARIES

The k-Fibonacci numbers which are of recent origin were found by studying the recursive application of two geometrical transformations used in the well-known four triangle longest-edge partition [10], serving as an example between geometry and numbers. Also in [18], authors established some new properties of k-Fibonacci numbers and k-Lucas numbers in terms of binomial sums. Falcon and Plaza [2] studied 3-dimensional k-Fibonacci spirals considering geometric point of view. Some identities for k-Lucas numbers may be found in [2]. In [4] many properties of k-Fibonacci numbers are obtained by easy arguments and related with so-called Pascal triangle.

In this section, we Review Basic Definitions and Introduce Relevant Facts

Definition (k-Fibonacci sequence [4]).: For any positive real number k, the k-Fibonacci sequence $\left\{F_{k,n}\right\}$ is defined by the recurrence relation

$$F_{k,n+1} = kF_{k,n} + F_{k,n-1}\ ,\ \ n \geq 1\ \ with\ F_{k,0} = 0,\ F_{k,1} = 1 \tag{2.1}$$

A few k-Fibonacci numbers are

$$F_{k,2} = k$$

$$F_{k,3} = k^2 + 1$$

$$F_{k,4} = k^3 + 2k$$

$F_{k,5} = k^4 + 3k^2 + 1 \ldots$

There is a large number of k-Fibonacci sequences indexed in The Online Encyclopedia of Integer Sequences, from now on OEIS, being the first

$\{F_{1,n}\} = \{0, 1, 1, 2, 3, 5, 8, \ldots\}$: A000045

$\{F_{2,n}\} = \{0, 1, 2, 5, 12, 29, \ldots\}$: A000129

$\{F_{3,n}\} = \{0, 1, 3, 10, 33, 109, \ldots\}$: A006190

Preposition 2.1: (Binet's formula for the k-Fibonacci sequence [4]). The nth k-Fibonacci number is given by

$$F_{k,n} = \frac{\Re_1^n - \Re_2^n}{\Re_1 - \Re_2} \tag{2.2}$$

where $\Re_1$ & $\Re_2$ are the roots of the characteristic equation $x^2 - kx - 1 = 0$.

Definition (k-Lucas sequence [1]). For any positive real number k, the k-Lucas sequence $\{L_{k,n}\}$ is defined by the recurrence relation

$$L_{k,n+1} = kL_{k,n} + L_{k,n-1}\ ,\ n \geq 1\ \ with\ L_{k,0} = 2,\ L_{k,1} = k \tag{2.3}$$

A few k-Lucas numbers are

$L_{k,2} = k^2 + 2$

$L_{k,3} = k^3 + 3k$

$L_{k,4} = k^4 + 4k^2 + 2$

$L_{k,5} = k^5 + 5k^3 + 5k$

Now on OEIS, being the first

$\{L_{1,n}\} = \{2, 1, 3, 4, 7, 11, 18, 29, \ldots\}$: A000032

$\{L_{2,n}\} = \{2, 2, 6, 14, 34, 82, 198, 478, \ldots\}$: A002203

$\{L_{3,n}\} = \{2, 3, 11, 36, 119, 393, 1298, 4287, \ldots\}$: A006497

Preposition 2.2: (Binet's formula for the k-Lucas sequence [1]). The k-Lucas numbers are given by the formula

$$L_{k,n} = \Re_1^n + \Re_2^n \tag{2.4}$$

where $\Re_1$ & $\Re_2$ are the roots of the characteristic equation $x^2 - kx - 1 = 0$.

Definition (k-Fibonacci-Like sequence [9]).: For any positive real number k, the k-Fibonacci-Like sequence $\{S_{k,n}\}$ is defined by the recurrence relation

$$S_{k,n+2} = kS_{k,n+1} + S_{k,n} \ , \ \ n \geq 0 \ \ with \ S_{k,0} = 2, \ S_{k,1} = 2k$$

A few k-Fibonacci-Like numbers are

$$S_{k,2} = 2k^2 + 2$$

$$S_{k,3} = 2k^3 + 4k$$

$$S_{k,4} = 2k^4 + 6k^2 + 2$$

$$S_{k,5} = 2k^5 + 8k^3 + 6k$$

Now

$$\{S_{1,n}\} = \ \{2, \ 2, \ 4, \ 6, \ 10, \ 16, \ldots\}$$

$$\{S_{2,n}\} = \ \{2, \ 4, \ 10, \ 24, \ 58, \ 130, \ldots\}$$

Preposition 2.3: (Binet's formula for the k-Fibonacci-Like sequence [9]). The k-Fibonacci-Like numbers are given by the formula

$$S_{k,n} = 2\frac{\Re_1^{n+1} - \Re_2^{n+1}}{\Re_1 - \Re_2} \tag{2.6}$$

where $\Re_1$ & $\Re_2$ are the roots of the characteristic equation $x^2 - kx - 1 = 0$

IDENTITIES OF THE K-FIBONACCI-LIKE SEQUENCE

The There are a lot of identities of Fibonacci and Lucas numbers described in [8]. Thongmoon [16, 17], defined various identities of Fibonacci and Lucas numbers. Singh, Bhadouria and Sikhwal [13], present some generalized identities involving common factors of Fibonacci and Lucas numbers. Gupta and Panwar [5], present identities involving common factors of generalized Fibonacci, Jacobsthal and jacobsthal-Lucas numbers. Panwar, Singh and Gupta ([11, 12]), present Generalized Identities Involving Common factors of generalized Fibonacci, Jacobsthal and jacobsthal-Lucas numbers. Singh, Sisodiya and Ahmed [15], investigate some products of k-Fibonacci and k-Lucas numbers, also present some generalized identities on the products of k-Fibonacci and k-Lucas numbers to establish connection formulas between them with the help of Binet's formula . In this paper, we present identities involving common factors of k-Fibonacci-Like, k-

Fibonacci and k-Lucas numbers. Generalized k-Fibonacci-Like sequence [9], similar to the other second order classical sequences. In this section we present generalized identities involving common factors of generalized k-Fibonacci-Like, k-Fibonacci and k-Lucas numbers. We shall use Binet's formula for derivation.

Theorem 3.1: $S_{k,4n-1} + 2k = S_{k,2n-2} L_{k,2n+1} \quad , \; where \; n \geq 1$ (3.1)

Proof:

$$S_{k,2n-2} L_{k,2n+1} = \frac{2}{\Re_1 - \Re_2} \left(\Re_1^{2n-1} - \Re_2^{2n-1} \right) \left(\Re_1^{2n+1} + \Re_2^{2n+1} \right)$$

$$= \frac{2}{\Re_1 - \Re_2} \left\{ \left(\Re_1^{4n} - \Re_2^{4n} \right) + \left(\frac{\Re_2}{\Re_1} - \frac{\Re_1}{\Re_2} \right) \right\}$$

$$= \frac{2}{\Re_1 - \Re_2} \left(\Re_1^{4n} - \Re_2^{4n} \right) + 2 \left(\Re_1 + \Re_2 \right)$$

$$= S_{k,4n-1} + 2k$$

This completes the proof.

Corollary 3.2: $For \; n \geq 1, \quad S_{k,2n-2} L_{k,2n+1} = 2 \left[F_{k,4n} + k \right]$ (3.2)

Following theorems can be solved with the help of Binet's formula

Theorem 3.3: $S_{k,4n-2} + 2 = S_{k,2n-2} L_{k,2n} \quad , \; where \; n \geq 1$ (3.3)

Corollary 3.4: $For \; n \geq 1, \quad S_{k,2n-2} L_{k,2n} = 2 \left[F_{k,4n-1} + 1 \right]$ (3.4

Theorem 3.5: $S_{k,4n+3} + S_{k,3} = S_{k,2n+3} L_{k,2n} \quad , \; where \; n \geq 0$ (3.5)

Corollary 3.6: $For \; n \geq 0, \quad S_{k,2n+3} L_{k,2n} = 2 \left[F_{k,4n+4} + F_{k,4} \right]$ (3.6)

Theorem 3.7: $S_{k,4n+4} + 2 = S_{k,2n+2} L_{k,2n+2} \quad , \; where \; n \geq 0$ (3.7)

Corollary 3.8: $For \; n \geq 0, \quad S_{k,2n+2} L_{k,2n+2} = 2 \left[F_{k,4n+5} + 1 \right]$ (3.8)

Theorem 3.9: $S_{k,4n+2} - 2 = S_{k,2n+1} L_{k,2n+1} \quad , \; where \; n \geq 0$ (3.9)

Corollary 3.10: $For \; n \geq 0, \quad S_{k,2n+1} L_{k,2n+1} = 2 \left[F_{k,4n+3} - 1 \right]$ (3.10)

Theorem 3.11: $S_{k,4n} - 2 = S_{k,n-1} L_{k,n} L_{k,2n+1} \quad , \; where \; n \geq 1$ (3.11)

Corollary 3.12: $For\ n \geq 1,\ \ S_{k,n-1} L_{k,n} L_{k,2n+1} = 2\left[F_{k,4n+1} - 1\right]$ (3.12)

Theorem 3.13: $\left(k^2 + 4\right) S_{k,2n-1} S_{k,2n} = 4\left(L_{k,4n+1} - k\right),\ where\ n \geq 1$ (3.13)

Theorem 3.14: $\left(k^2 + 4\right) S_{k,2n+1} S_{k,2n} = 4\left(L_{k,4n+1} + k\right),\ where\ n \geq 0$ (3.14)

CONCLUSIONS

In this paper, we present many identities of common factors of k-Fibonacci-Like, k-Fibonacci and k-Lucas numbers with the help of their Binet's formula. The concept can be executed for generalized Fibonacci sequences as well as polynomials.

REFERENCES

1. Falcon, S., "On the k-Lucas Numbers," *Int. J. Contemp. Math. Sci.*, 6(21), 1039-1050, 2011.
2. Falcon, S., Plaza, A., "On the 3-dimensional k-Fibonacci spirals," *Chaos, Solitons and Fractals*, 38(4), 993–1003, 2008.
3. Falcon, S., Plaza, A., "On the Fibonacci k-numbers," *Chaos, Solitons and Fractals*, 32(5), 1615–1624, 2007.
4. Falcon, S., Plaza, A., "The k-Fibonacci sequence and the Pascal 2-triangle," *Chaos, Solitons and Fractals*, 33(1), 38–49, 2007.
5. Gupta, V. K., Panwar, Y. K., "Common Factors of Generalized Fibonacci, Jacobsthal and Jacobsthal-Lucas numbers," *International Journal of Applied Mathematical Research*, 1(4), 377-382, 2012.
6. Horadam, A. F., "The Generalized Fibonacci Sequences," *The American Math. Monthly*, 68(5), 455-459, 1961.
7. Kalman, D., Mena, R., "The Fibonacci Numbers – Exposed," *The Mathematical Magazine*, 2(2002).
8. Koshy, T., "Fibonacci and Lucas Numbers with Applications," *Wiley-Interscience Publication*, New York, 2001.
9. Panwar, Y. K., Rathore, G. P. S., Chawla, R., "On the k-Fibonacci-Like Numbers," *Turkish J Analysis Number Theory*, 2(1), 9-12, 2014.
10. Panwar, Y. K., Singh, B., Gupta, V. K., "Generalized Fibonacci Sequence and Its properties," *Palestine J. Math.*, 3(1), 141-147, 2014.
11. Panwar, Y. K., Singh, B., Gupta, V. K., "Generalized Identities Involving Common Factors of Generalized Fibonacci, Jacobsthal and Jacobsthal-Lucas numbers," *International journal of Analysis and Application*, 3(1), 53-59, 2013.
12. Panwar, Y. K., Singh, B., Gupta, V. K., "Identities of Common Factors of Generalized Fibonacci, Jacobsthal and Jacobsthal-Lucas Numbers." *Applied Mathematics and Physics* 1(4), 126-128, 2013.
13. Singh, B., Bhadouria, P., Sikhwal, O., "Generalized Identities Involving Common Factors of Fibonacci and Lucas Numbers," *International Journal of Algebra*, 5(13), 637-645, 2013.

14. Singh, B., Sikhwal, O., Bhatnagar, S., "Fibonacci-Like Sequence and its Properties," *Int. J. Contemp. Math. Sci.*, 5(18), 859-868, 2010.

15. Singh, B., Sisodiya, K., Ahmed, F., "On the Products of k-Fibonacci Numbers and k-Lucas Numbers," *International Journal of Mathematics and Mathematical Sciences*, 2014, Article ID 505798, 4 pages . http://dx.doi.org/10.1155/2014/505798

16. Thongmoon, M., "Identities for the common factors of Fibonacci and Lucas numbers," *International Mathematical Forum*, 4(7), 303–308, 2009.

17. Thongmoon, M., "New identities for the even and odd Fibonacci and Lucas numbers," *International Journal of Contemporary Mathematical Sciences*, 4(14), 671–676, 2009.

18. Yilmaz, N., Taskara, N., Uslu, K., and Yazlik, Y., "On the binomial sums of k-Fibonacci and k-Lucas sequences," *in Proceedings of the International Conference on Numerical Analysis and Applied Mathematics (ICNAAM '11)*, 341–344, September 2011.