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A DETERIORATING INVENTORY MODEL WITH LINEAR, EXPONENTIAL DEMAND WITH SHORTAGE

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ABSTRACT

We study a two type of demand situations inventory model for deteriorating items. First type the demand pattern follows linearly increase with time and secondly it follow exponential increase with time and finally shortage are allowed. Numerical example is used to illustrate the developed model. Sensitivity analysis of the optimal solution with respect to major parameter is carried out.

KEYWORDS: *Inventory, Deterioration, Demand, Shortage*

INTRODUCTION

In real life, the harvest of food grains like paddy, wheat, and fruit like mango etc. is periodic. As there are a large number of landless people in the rural areas of India, there will be a constant demand for these food grains throughout the year. Due to various reasons, some of the farmers are forced to sell some part of their food grains and as a result, they buy the food grains from the market towards the end of the production cycle. Therefore, the rate of demand for food grains remains partly constant and increases partly with time.

The demand pattern assumed here occurs not only for seasonal product, but also for fashion apparel, computer chips of the advanced computer, spare parts, etc. The nature of demand for seasonal and fashionable products increasing then steady then decreasing and finally vanishes. The demand of the item increases with time and then stabilizers after some time and ultimately becomes constant. Thus the demand rate is deterministic when any new brand product is launched in the markets, the demand rate linearly depends on time, and later it gets stabilized in the market.

Deterioration is defined as decay, damage, spoilage, evaporation, or drying out of products. Thus, the ideal case envisioned by the classical model remains an ideal one. The effects of deterioration are significant in many inventories systems, making the problem of how to control and maintain inventories of deteriorating items a major issue for decision makers in modern organizations. In addressing this issue, Ghare and Schrader (1963) first proposed a model for an exponentially decaying inventory. Sahu et al.(2006),Sethi and Sahu (2007),Sahu et al. (2007),Samal et al.(2008), Kalam et al.(2008),Sahu and Sukla(2008), Begum et al, (2009) developed a model with exponential demand for finite production rate and shortages. There exist several inventory models that take into account the constant deterioration rate, Weibull deterioration rate, and linear time fluctuating demand. These inventory models were developed by Goswami and Chaudhuri (1991), Chakrabarti and Chaudhuri (1997), Giri et al. (2000),Singh et al.(2012), Yang (2010), Skouri et al.

(2011), Begum et al.(2009),Samal et al.(2009),Shah (2015), Taleizadeh et al.(2016). Jalan and Chaudhuri (1999), Sahu and Dash (2006) developed inventory models with constant deterioration, constant demand, and instantaneous replenishment. The inventory models with a fixed deteriorating rate and an exponential time fluctuating demand and these were considered by Wee (1995), and Hariga and Benkherouf (1994). The situation of deterioration and permissible shortages is developed by Jamal et al. (1997) and Chang and Dye (2001). Hsu et al. (2010) built a deterioration inventory model considering constant demand, deterioration rate, and preservation technology investment that reduces the deterioration rate of products. A numerous number of researchers have investigated on inventory models with constant demand rate or time-varying demand patterns. A few of the researchers like Barbosa and Friedman (1978), Data and Pal (1988, 1990), Urban (1992), Urban and Baker (1997), Ray et al.(1998),.Donaldson (1977) first developed an exact solution procedure for items with a linearly increasing demand rate over a finite planning horizon.

In this paper, we study a two type of demand situations inventory model for deteriorating items. The first type the demand pattern follows linearly increase with time and secondly it follow an exponential increase with time and finally shortage occurred. The demand of a product may increase with time due to the incoming of a new product like vegetables and fruits, which may be technically good and attractive than old one, and also the demand of the new product may increase, decrease with time. In a real market situation, demand is unlikely increases at a rate, which is very high as linear and exponential.

NOTATION & ASSUMPTION

θ Deterioration rate.

h Inventory holding cost per item per unit time.

s Shortage cost per item per unit time.

A Set-up cost per cycle.

p Purchasing cost per unit item.

c Time horizon.

d Deteriorating cost per unit time.

r Revenue cost per unit time.

o Ordering cost per unit time.

The inventory system is based on the following assumptions:

1.A single item is considered in the inventory system.

2.Replenishment rate is finite

3. Lead time is zero

4.Shortages are allowed.

5.The demand rate at first time period follow linear $D(t) = \alpha + \beta t, \quad \alpha, \beta > 0$ Secondly the demand $D(t) = \beta e^{\beta t}$, $\alpha, \beta \geq 0$

6.The deterioration rate of items is constant.

MODEL FORMULATION

At the start of the cycle, the inventory level is maximum units of items at time $t=0$.During the time interval$[0,a]$ inventory depletes due to mainly demand and no deterioration. At time $t=a$, the inventory depletes due to exponential demand and time $t=b$ the inventory level is zero. The change in inventory at any time t are governed by the following differential equations:

$$\frac{dI[t]}{dt}=-\alpha-\beta t \qquad 0\le t\le a \tag{1}$$

With boundary condition $I[0]=Q$

$$\frac{dI[t]}{dt}+\theta I[t]=-\beta e^{\beta t} \quad a<t<b \tag{2}$$

Where $0<\theta<1$,With boundary condition $I[b]=0$

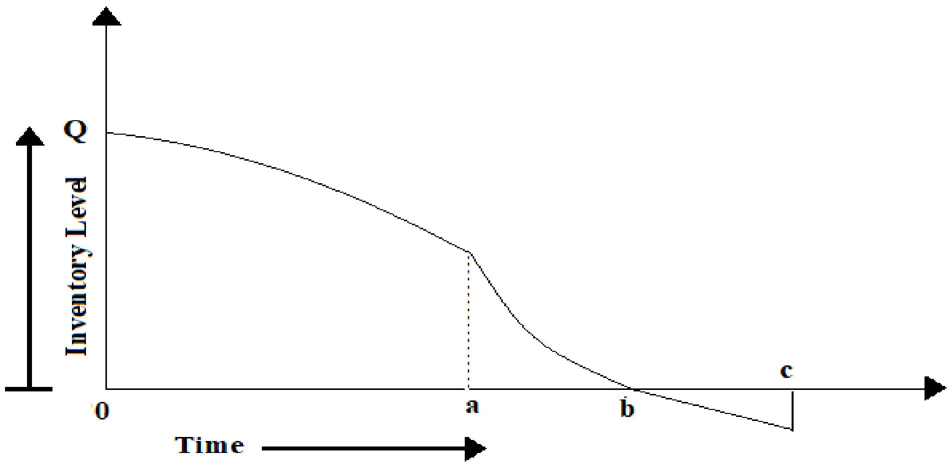

Figure 1:The system of inventory with linear demand ,exponential demand Pattern and shortage

$$\frac{dI[t]}{dt}=-\alpha \qquad b<t<c \tag{3}$$

With boundary condition $I[b]=0$

The solution of equation (1),(2) & (3) are

$$I[t]=\frac{1}{2}\left(2Q-2t\alpha-t^2\beta\right) \qquad 0\le t\le a \tag{4}$$

$$I[t]=\frac{e^{-t\theta}\left(e^{b(\beta+\theta)}-e^{t(\beta+\theta)}\right)\beta}{\beta+\theta}, \quad a<t<b \tag{5}$$

$$I[t]=b\alpha-\alpha t\ , \quad b<t<c \tag{6}$$

(A) Holding Cost(HC) $= h\int_0^b I(t)dt$

$$= h\left[aQ - \frac{a^2\alpha}{2} - \frac{a^3\beta}{6} + \frac{e^{-a\theta+b(\beta+\theta)} + \theta e^{a\beta} - (\beta+\theta)e^{b\beta}}{\beta\theta(\beta+\theta)}\right]$$

(B) Shortage Cost(SC) $= s\int_b^c I(t)dt$

$$= s\left[\frac{b^2\alpha}{2} - bc\alpha + \frac{c^2\alpha}{2}\right]$$

(C) Deteriorating Cost (DC) $= \frac{1}{2}\left(-a^2 + b^2 + \frac{2\left(e^{a\beta}(-1+a\beta) + e^{b\beta}(1-b\beta)\right)}{\beta}\right)$

(D) Ordering Cost (OC) $= A$

(E) Purchasing Cost(PC) $= p\left(Q - b\alpha + c\alpha\right)$

(F) Sales Revenue (SR)= $= r\left(a\alpha + \frac{a^2\beta}{2} - e^{\alpha\beta} + e^{b\beta} - b\alpha + c\alpha\right)$

(G) Total average profit G =

$$G(\alpha,\beta,a,b,c,h,r,s,p,Q,e,o,A,d) = \frac{SR - (OC + HC + SC + PC + DC)}{c}$$

$$= r\left(a\alpha + \frac{a^2\beta}{2} - e^{\alpha\beta} + e^{b\beta} - b\alpha + c\alpha\right) + h\left(aQ - \frac{a^2\alpha}{2} - \frac{a^3\beta}{6} + \frac{e^{-a\theta+b(\beta+\theta)} + \theta e^{a\beta} - (\beta+\theta)e^{b\beta}}{\beta\theta(\beta+\theta)}\right) - s\left(\frac{b^2\alpha}{2} - bc\alpha + \frac{c^2\alpha}{2}\right) - p\left(Q - b\alpha + c\alpha\right)$$
$$- \frac{d}{2}\left(-a^2 + b^2 + \frac{2\left(e^{a\beta}(-1+a\beta) + e^{b\beta}(1-b\beta)\right)}{\beta}\right) - oA$$

The necessary condition for optimality of $G(\alpha,\beta,a,b,c,h,r,s,p,Q,e,o,A,d)$ is

$$\partial_{G_a} = 0$$

$$\partial_{G_b} = 0$$

$$\partial_{G_c} = 0$$

EXAMPLE

We consider the parameter lues $\alpha = 10$, $\beta = 0.1$, $h = 0.2$, $r = 5$, $s = 2$, $p = 3$, $Q = 100$, $\theta = 0.3$, $e \approx 2.8$,
$A = 100$, $o = 4$, $d = 1$,then

$$G = -300 + 29.99a + 1.25a^2 + 0.0033a^3 - 10b^2 + 20c - 10c^2 - 1.66e^{-0.3a+0.4b}$$
$$+11.66e^{0.1b} + 20b(c-1) - 10e^{0.1a}$$

$$\partial_{G_a} = 15 - +2.25a + 0.005a^2 - 0.1(a-10)e^{0.1a} - 1.25e^{-0.3a+0.4b} = 0 \quad \text{(i)}$$

$$\partial_{G_b} = -21 - 21b - 20c - 0.67e^{-0.3a+0.4b} + 0.1(b+8.2)e^{0.1b} = 0 \quad \text{(ii)}$$

$$\partial_{G_c} = 20 + 20b - 20c = 0 \quad \text{(iii)}$$

By solving the equation (i),(ii) & (iii) using Mathematica 9, we get

$$a = 6.74, b = 9.54, c = 10.03, G = 481.67$$

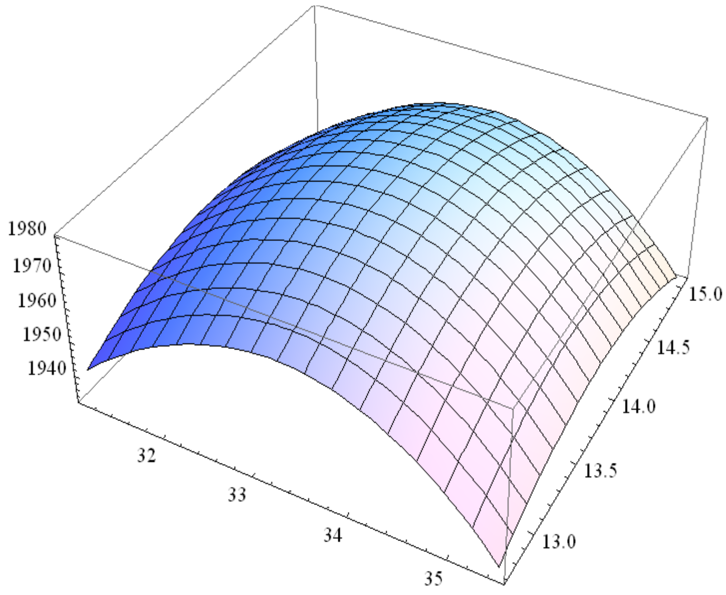

Figure 2: Total Inventory Cost for the above Example

SENSITIVITY ANALYSIS

Effect on G by changing of % value of parameters

Table 1: Change of value of 'α'

Original Value	% of changes	Changed Value	Ratio of changed values to original values G
$\alpha = 10$	+50	15	+2.394
$\alpha = 10$	+25	12.5	+1.698
$\alpha = 10$	-25	07.5	+0.302
$\alpha = 10$	-50	05	-0.395

Table 2: Change of value of 'β'

Original Value	% of changes	Changed Value	Ratio of changed values to original values G
$\beta = 0.1$	+50	0.05	+0.982
$\beta = 0.1$	+25	0.025	+0.989
$\beta = 0.1$	-25	0.125	+1.004
$\beta = 0.1$	-50	0.15	+1.008

Table 3: Change of value of 'θ'

Original Value	% of changes	Changed Value	Ratio of changed values to original values G
$\theta = 0.3$	+50	0.45	+0.998
$\theta = 0.3$	+25	0.375	+0.999
$\theta = 0.3$	-25	0.225	+1.001
$\theta = 0.3$	-50	0.15	+1.002

Table 4: Change of value of 'h'

Original Value	% of changes	Changed Value	Ratio of changed values to original values G
$h = 0.2$	+50	0.3	+0.905
$h = 0.2$	+25	0.25	+0.949
$h = 0.2$	-25	0.15	+1.046
$h = 0.2$	-50	0.1	+1.095

Table 5: Change of value of 'r'

Original Value	% of changes	Changed Value	Ratio of changed values to original values G
$r = 5$	+50	7.50	+2.357
$r = 5$	+25	6.25	+1.678
$r = 5$	-25	3.75	+0.321
$r = 5$	-50	2.50	-0.357

Table 6: Change of value of 's'

Original Value	% of changes	Changed Value	Ratio of changed values to original values G
$s = 2$	+50	3	+0.997
$s = 2$	+25	2.5	+0.998
$s = 2$	-25	1.5	+1.001
$s = 2$	-50	1	+1.003

Table 7: Change of value of 'p'

Original Value	% of changes	Changed Value	Ratio of changed values to original values G
$p = 3$	+50	4.5	+0.673
$p = 3$	+25	3.75	+0.837
$p = 3$	-25	2.25	+1.163
$p = 3$	-50	1.5	+1.327

Table 8: Change of value of $'o'$

Original Value	% of changes	Changed Value	Ratio of changed values to original values G
$o = 4$	+50	6	+0.584
$o = 4$	+25	5	+0.792
$o = 4$	-25	3	+1.207
$o = 4$	-50	2	+1.415

Table 9: Change of value of $'A'$

Original Value	% of changes	Changed Value	Ratio of changed values to original values G
$A = 100$	+50	150	+0.584
$A = 100$	+25	125	+0.792
$A = 100$	-25	75	+1.207
$A = 100$	-50	50	+1.415

Table 10: Change of value of $'d'$

Original Value	% of changes	Changed Value	Ratio of changed values to original values G
$d = 1$	+50	1.5	+0.981
$d = 1$	+25	1.25	+0.991
$d = 1$	-25	0.75	+1.009
$d = 1$	-50	0.50	+1.018

Table 11: Change of value of $'Q'$

Original Value	% of changes	Changed Value	Ratio of changed values to original values G
$Q = 100$	+50	150	+0.548
$Q = 100$	+25	125	+0.774
$Q = 100$	-25	75	+1.225
$Q = 100$	-50	50	+1.451

By increasing the value of "α" by 50% ,25%,the optimum value "G" changed by 2.393,1.698 times respectively from its original values. Similarly by decreasing the value of "α" by 25%, 50%, the optimum value "G" changed by 0.302,-0.395 times respectively from its original values. Finally we conclude that by increasing the value of "α", value of "G" increasing and by decreasing the value of "α" , value of "G" decreasing.

By increasing the value of "β" by 50% ,25%,the optimum value "G" changed by 0.982,0.989 times respectively from its original values. Similarly by decreasing the value of "β" by 25%, 50%, the optimum value "G" changed by1 .004, 1.008 times respectively from its original values. Finally we conclude that by increasing the value of "β", value of "G" decreasing and by decreasing the value of "β", value of "G" increasing.

By increasing the value of "θ" by 50% ,25%,the optimum value "G" changed by 0.998 , 0.999 times respectively from its original values .Similarly by decreasing the value of "θ" by 25% ,50%,the optimum value "G" changed by 1.001, 1.002 times respectively from its original values. Finally we conclude that by increasing the value of " θ ", value of "G" decreasing and by decreasing the value of " θ ", value of "G" increasing.

By increasing the value of " h " by 50% ,25%,the optimum value "G" changed by 0.905, 0.949 times respectively from its original values. Similarly by decreasing the value of " h " by 25%, 50%, the optimum value "G" changed by 1.046, 1.095 times respectively from its original values. Finally we conclude that by increasing the value of " h ", value of "G" decreasing and by decreasing the value of " h ", value of "G" increasing.

By increasing the value of " r " by 50% ,25%,the optimum value "G" changed by 2.357,1.678 times respectively from its original values. Similarly by decreasing the value of " r "by 25%, 50%,the optimum value "G" changed by 0.321,-0.357 times respectively from its original values. Finally we conclude that by increasing the value of " r ", value of "G" increasing and by decreasing the value of " r ", value of "G" decreasing.

By increasing the value of " s " by 50% ,25%,the optimum value "G" changed by 0.997 , 0.998 times respectively from its original values. Similarly by decreasing the value of " s " by 25%, 50%, the optimum value "G" changed by 1.001, 1.003 times respectively from its original values. Finally we conclude that by increasing the value of " s ", value of "G" decreasing and by decreasing the value of " s ", value of "G" increasing.

By increasing the value of " p " by 50% ,25%, the optimum value "G" changed by 0.673, 0.837 times respectively from its original values. Similarly by decreasing the value of " p "by 25%, 50%, the optimum value "G" changed by 1.163, 1.327 times respectively from its original values. Finally we conclude that by increasing the value of " p ", value of "G" decreasing and by decreasing the value of " p ", value of "G" increasing.

By increasing the value of " o " by 50% ,25%,the optimum value "G" changed by 0.584, 0.792 times respectively from its original values. Similarly by decreasing the value of " o " by 25%, 50%, the optimum value "G" changed by 1.207, 1.415 times respectively from its original values. Finally we conclude that by increasing the value of " o ", value of "G" decreasing and by decreasing the value of " o ", value of "G" increasing.

By increasing the value of " A " by 50% ,25%,the optimum value "G" changed by 0.584, 0.792 times respectively from its original values. Similarly by decreasing the value of " A " by 25%, 50%, the optimum value "G" changed by 1.207, 1.415 times respectively from its original values. Finally we conclude that by increasing the value of " A ", value of "G" decreasing and by decreasing the value of " A ", value of "G" increasing.

By increasing the value of " d " by 50%, 25%, the optimum value "G" changed by 0.981, 0.991times respectively from its original values. Similarly by decreasing the value of " d "by 25%, 50%, the optimum value "G" changed by 1.009, 1.018 times respectively from its original values. Finally we conclude that by increasing the value of "d", value of "G" decreasing and by decreasing the value of "d", value of "G" increasing.

By increasing the value of "Q" by 50% ,25%,the optimum value "G" changed by 0.548, 0.774 times respectively from its original values. Similarly by decreasing the value of "Q"by 25%, 50%, the optimum value "G" changed by 1.225, 1.451 times respectively from its original values. Finally we conclude that by increasing the value of "Q", value of "G" decreasing and by decreasing the value of "Q", value of "G" increasing

CONCLUSION

The demand of a product may increase with time due to the incoming of a new product like vegetables and fruits which may be technically good and attractive than old one, and also the demand of the new product may increase, decrease with time. In a real market situation, demand is increases at a rate, which is very high as linear and exponential. Whenever some new attractive products launched in super market or some seasonal items happen in beginning of season like winter, the demand of that product or item is increasing depending upon rate of purchase. This type of demand is quite appropriate for products like winter vegetables, fruits in the city of Himachal Pradesh, Jammu-Kashmir and summer vegetables, fruits in the city of Odisha like Berhampur especially mango market, Rourkela market etc. As the season progress, the demand rate goes on increasing and gradually approaching a saturation level and finished.

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